NGA West 2 Estimation of Epistemic Uncertainty

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Components of Epistemic Uncertainty

1. Model to model differences
   - Represented by 5 NGA models
2. Uncertainty in predictions from an individual GMPE
   - Can be quantified statistically
3. Other uncertainty not covered by 1 & 2

Model to Model Component

- Measured by computing standard deviation of GMPE predictions as a function of **M** and R

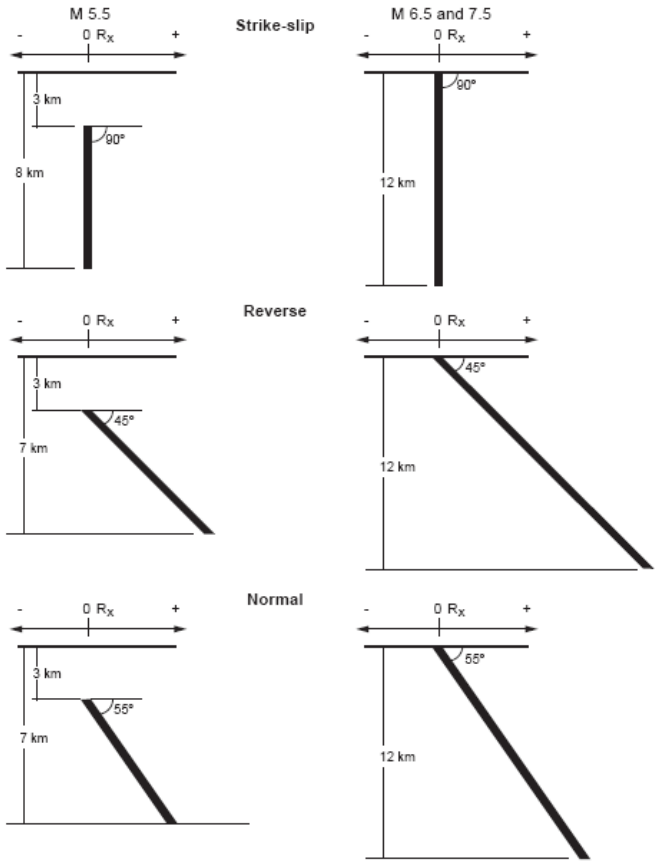
$$\sigma_{\mu \ln(SA|M,R)} = \sqrt{\frac{\sum_i w_i \left[\mu_{\ln(SA|M,R)_i} - \overline{\mu_{\ln(SA|M,R)}}\right]^2}{\sum_i w_i}}$$

with

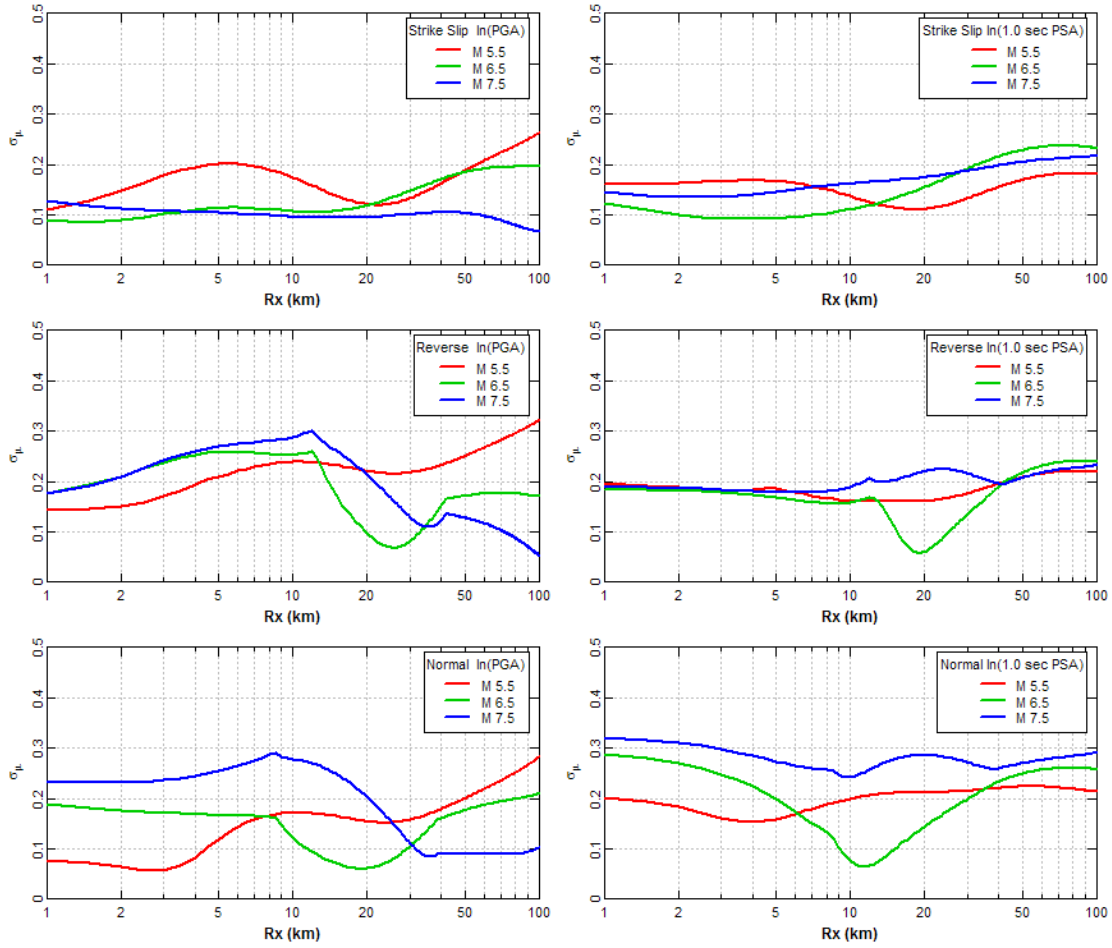
$$\overline{\mu_{\ln(SA|M,R)}} = \frac{\sum_i w_i \mu_{\ln(SA|M,R)_i}}{\sum_i w_i}$$

Example Calculations

- Set up a set of scenarios for **M**, R, and faulting type
- Compute median values for NGA GMPEs, $\mu_{(\ln SA|M,R)i}$
- Computed standard error of these median estimates



Model to Model σ_μ



Uncertainty in Median Prediction for a Single GMPE

- Simple linear model

$$y = a + bx$$

- Uncertainty in mean of *y* given a new value x_0

$$\sigma^2_{\bar{y}|x_0} = \frac{\sigma^2}{n}\left[1 + \frac{(x_0 - \bar{x})}{\sigma_x^2}\right]$$

- GMPE

$$\sigma^2_{\overline{\ln(y)}|x_0} = \mathbf{f}^T\left[\mathbf{F}^T\mathbf{V}^{-1}\mathbf{F}\right]^{-1}\mathbf{f}$$

$$\mathbf{F} = \left.\frac{\partial\overline{\ln(y)}}{\partial\mathbf{C}}\right|_{\mathbf{x}_i}$$

Gradient of model with respect to coefficients **C** evaluated at data $\mathbf{x}_i$ used in regression

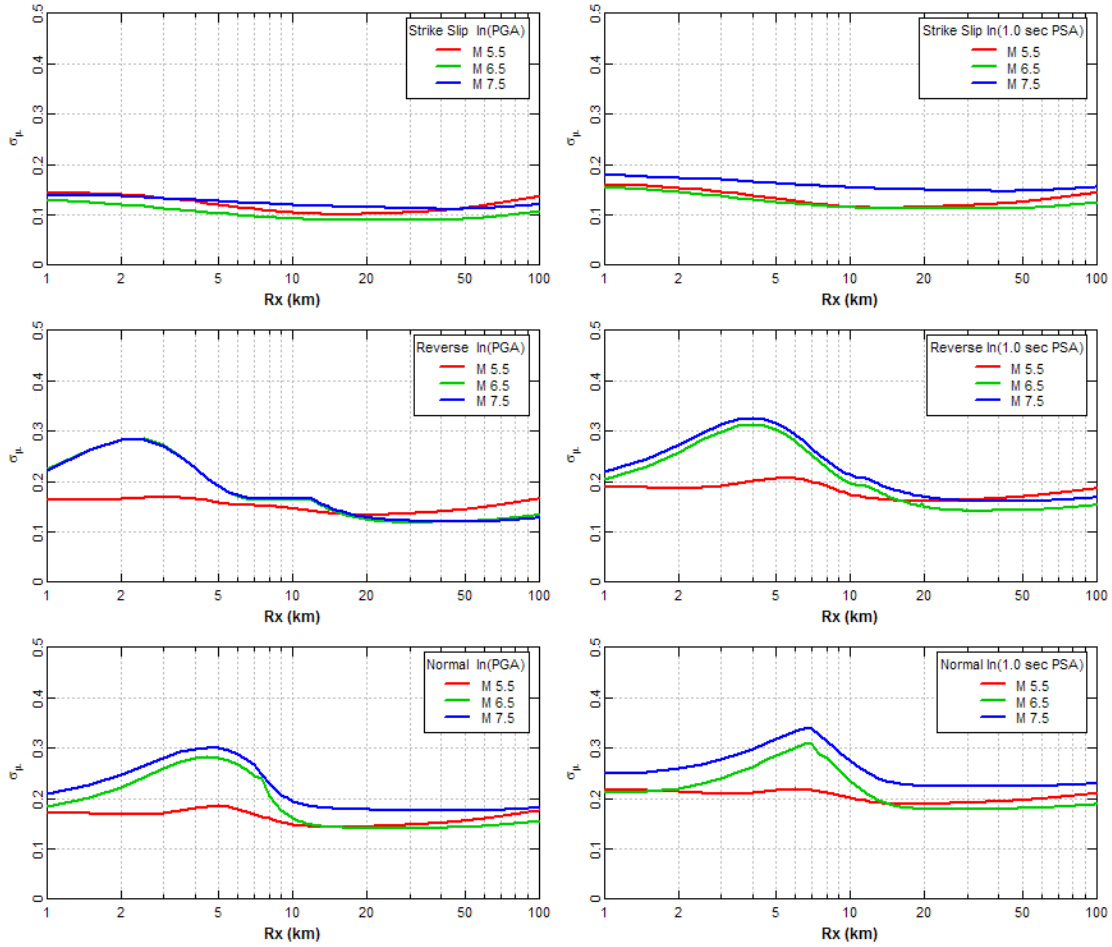
$$\mathbf{f} = \left.\frac{\partial\overline{\ln(y)}}{\partial\mathbf{C}}\right|_{\mathbf{x}_0}$$

Gradient of model with respect to coefficients **C** evaluated at new data $\mathbf{x}_0$ used for prediction

$\mathbf{V}$

Block diagonal variance matrix

Example Calculations for Chiou and Youngs (2008)



NGA West 2 Epistemic Model

- Compute variance of model predictions for each of the NGA GMPEs
  - Begin with NGA 2008
  - Repeat with final NGA 2012
- Provide recommended model to represent epistemic uncertainty in individual NGA GMPEs for inclusion in final composite epistemic uncertainty recommendation from developers